

## Research Paper

## Spectrum Sensing in Cognitive Radios Using Co-Variance Matrix.

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**Abstract:** Spectrum sensing is one of the most important components in any cognitive radio system that allows usage of the underutilized portions of the radio spectrum. First worldwide standard for cognitive radios to operate in the recently available TV bands is being formulated by IEEE 802.22 Working Group. For this standard to succeed, it is necessary that the presence of TV signals is detected using a reliable sensing mechanism. Spectrum sensing algorithm using statistical covariance and its variants have attracted lot of attention recently. Spectrum sensing algorithms based on statistical covariance have advantages over the conventional approach based on energy detection. Covariance based signal detection does not require apriority knowledge of noise power. Spectral covariance of the received signal for signal detection was proposed<sup>(2)</sup>. The proposed algorithm exploits statistical correlation of the signal, in particular the pilot signal in frequency domain. Detection performance of this technique shows improved sensitivity compared to other pilot detection algorithms. The properties of the eigenvalue of the covariance matrix have also been used to detect the presence of radio signal. Random matrix theory has been employed to derive the probability of false alarm and probability of missed detection for eigenvalue based signal sensing. In this paper we propose to discuss different spectrum sensing approaches based on statistical covariance matrix and compare the performance metrics of each approach. Further, we will also suggest techniques to improve the performance of covariance based spectrum sensing approach. Techniques to apply these algorithms to a wider class of signals will also be discussed in this paper.

**Key Words:** Spectrum, sensing, detection, eigenvalues, cognitive radio.

## 1. Introduction

Explosive growth in the number of wireless devices operating in the unlicensed as well licensed bands has resulted in severe shortage of radio spectrum. The multitude of wireless networks and protocols (e.g., Wi-Fi, Bluetooth, WiMAX etc.) Operating in the unlicensed bands and vying for their share of the spectrum has led to interference and performance degradation for all the users. However, recent studies by the Federal Communication Commission (FCC) in US and OFCOM have shown that at any given time and in any given geographic locality, less than 10% of the available spectrum in the licensed band is utilized. To exploit these underutilized parts of the spectrum (also referred to as white spaces or spectrum holes), the FCC has advocated development of a new generation of programmable, smart radios that can dynamically access various parts of the spectrum, including the licensed bands. Such radios would operate as secondary users in the licensed bands. These radios are required to possess the capabilities of spectrum usage sensing, environment learning and interference avoidance with the primary users of the licensed spectrum bands while simultaneously ensuring

the quality of service ( $QoS$ ) requirements of both the primary and secondary users. Radios with such capabilities are referred to as cognitive radios (CRs)

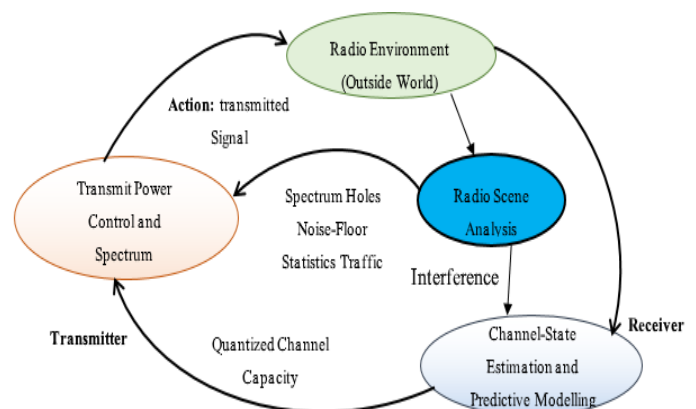


Figure -1: Basic Cognitive Cycle

## 2. System (Signal) Model

Assume that the frequency band of interest has a central frequency  $f_c$  and bandwidth  $W$ . During a particular time interval, the frequency band may be occupied by only one primary user. Several secondary users are randomly distributed in the cognitive radio network. Each secondary user is equipped with a single antenna. In this research work, the non-cooperative spectrum sensing scheme is considered, that is, the sensing work is completed by only one secondary user (only one source, one receiver). For signal detection, two hypotheses can be formulated:

- hypothesis  $H_0$  : there exists no signal (only noise);
- Hypothesis  $H_1$ : there exists both the signal and additive white noise. The binary hypothesis test can be replaced by:  $H_0 : x(n) = w(n)$ ,  $n = 0, 1, \dots$

In a single radio based sensing approach, even the weak signals must be detected to avoid causing interference to primary receivers within its transmission zone. The basic hypothesis problem for transmitter detection is usually formulated as:

$$H_1: x(n) = \sum_{k=0}^n h(k)s(n-k) + w(n) \quad (1)$$

Where  $x(n)$  denotes the discrete signal at the secondary receiver,  $s(n)$  is the primary signal seen at the receiver,  $h(k)$  is the channel response,  $N$  is the order of the channel, and  $w(n)$  are the noise samples. Considering a sub-sample  $M$  of consecutive outputs and defining

$$\hat{X}(n) = [x(n), x(n-1), \dots, x(n-M+1)]^T$$

$$\hat{W}(n) = [w(n), w(n-1), \dots, w(n-M+1)]^T$$

$$\hat{S}(n) = [s(n), s(n-1), \dots, s(n-M+1)]^T$$

This result in

$$\hat{X}(n) = H\hat{S}(n) + \hat{W}(n) \quad (2)$$

Where

$H$  is an  $M \times (N+M)$  matrix, defined as

$$H = \begin{pmatrix} h(0) & \dots & h(N) & \dots & 0 \\ 0 & \dots & h(0) & \dots & h(N) \end{pmatrix} \quad (3)$$

Considering the statistical properties of the transmitted signal and channel noise, assume that the noise is white and that the noise and the transmitted signal are correlated. Let  $R$  be the covariance matrix of the received signal, that is,

$$R = 1/N_s \sum_{n=m}^{M-1+N_s} X(n)X^H(n) \quad (4)$$

Where  $N_s$  is the number of collected samples. If  $N_s$  is large, based on the assumptions made earlier, it is verified that

$$R \approx E[X(n)X^H(n)] = H R_s H^H + \sigma_w^2 I_M \quad (5)$$

Where  $R_s$  is the statistical covariance matrix of the input signal,  $R_s = E[s^*(n)s^H(n)]$ ,  $\sigma_w^2$  is the variance of the noise, and  $I_M$  denotes an  $M \times M$  identity matrix.

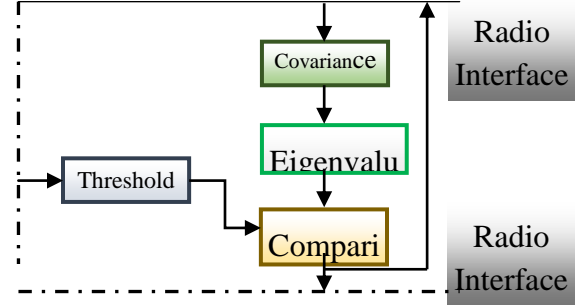


Figure - 2: Eigenvalue-Based Spectrum Sensing Algorithm Flow Chart

Let  $\lambda^{\max}$  and  $\lambda^{\min}$  be the maximum and the minimum eigenvalues of  $R$ , and  $\rho^{\max}$  and  $\rho^{\min}$  be the maximum and the minimum eigenvalues of  $H R_s H^H$ . Then

$$\Lambda_{\max} = \rho_{\max} + \sigma_w^2 \quad \text{and}$$

$$\Lambda_{\min} = \rho_{\min} + \sigma_w^2 \quad (6)$$

obviously,  $\rho^{\max} = \rho^{\min}$  if and only if  $H R_s H^H = \delta I_M$  where  $\delta$  is a positive number.

Again, obviously, when the primary signal is present

$$\Lambda_{\max} = \rho_{\max} + \sigma_w^2 \quad \text{and} \quad \Lambda_{\min} = \sigma_w^2$$

And when the primary Signal is absent then  $\Lambda_{\max} = \Lambda_{\min} = \sigma_w^2$

Hence, if there is no signal,  $\lambda^{\max}/\lambda^{\min} = 1$ ; otherwise  $\lambda^{\max}/\lambda^{\min} > 1$

The ratio of  $\lambda^{\max}/\lambda^{\min}$  can be used to detect the presence of the primary signal. However,  $\lambda^{\max}$  and  $\lambda^{\min}$  are the estimated eigenvalues.

### 2.1. Detection Algorithm Flow Chart

Figure 2 illustrates the main parts of the proposed method. The sampled signal comes from the radio system interface, from which the covariance matrix is built. The eigenvalues of the matrix are the calculated with a specific algorithm to form a maximum-minimum ratio; with the users threshold settings defined and signal presence detection done through comparison with the eigenvalues ratio.

### 2.2. Eigen-analysis of the Auto covariance Matrix

To better explain the detection algorithm, the eigenvalues of the auto covariance matrix is necessary. It is assumed that the random process  $x(n)$  is, in a wide-sense, stationary and its linear combinations of  $m$  basic components  $S_i(n)$  is given by

$$x(n) = \sum_{i=1}^m a_i S_i(n) \quad (7)$$

Since the equation observed is  $y(n) = x(n) + w(n)$ , where  $w(n)$  is a complex additive white Gaussian noise sequence with spectral density  $\sigma_w^2$ , the  $M \times M$  auto covariance matrix for  $y(n)$  can be expressed as

$$C_{yy} = C_{xx} + \sigma_w^2 I \quad (8)$$

Where  $C_{xx}$  is the autocovariance matrix for the signal  $x(n)$ ,  $\sigma_w^2 \mathbf{I}$  is the autocovariance matrix for the noise and  $M$  is the length of the covariance matrix. Note that if  $M > m$ , then  $C_{xx}$  which is of dimension  $M \times M$  is not of full rank.

Now, an eigen-decomposition of the matrix  $C_{yy}$  is performed. Let the eigenvalues be ordered in decreasing value with  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_M$  and let the corresponding eigenvectors be denoted as  $v_i, i = 1, \dots, M$ . It is assumed that the eigenvectors are normalized so that  $V_i^H V_j = \delta_{ij} (H)$ .

Where  $H$  denotes the Conjugate of the transpose In the absence of noise, the eigenvalues  $\lambda_i, i = 1, 2, \dots, m$  are nonzero while  $\lambda_{m+1} = \lambda_{m+2} = \dots = \lambda_M = 0$ . Thus, the eigenvectors  $v_i, i = 1, 2, \dots, m$  span the signal subspace. These vectors are called principal eigenvectors and the corresponding eigenvalues are called principal eigenvalues. In the presence of noise, the Eigen-decomposition separates the eigenvectors in two sets. The set  $v_i, i = 1, 2, \dots, m$ , which are the principal eigenvectors, span the signal subspace, while the set  $v_i, i = m + 1, \dots, M$ , which are orthogonal to the principal eigenvectors, are said to belong to the noise subspace. It follows that the signal  $x(n)$  is simply linear combinations of the principal eigenvectors. Finally, the variances of the projections of the signal on the principal eigenvectors are equal to the corresponding eigenvalues of the covariance matrix. So, the principal eigenvalues are the power factors in the new signal space. In the next subsection, the real maximum eigenvalue  $\lambda^{\max}$  and minimum eigenvalue  $\lambda^{\min}$  of the covariance matrix of the received signal will be obtained.

### 2.3. Power Method

In this section, the power method is exploited in order to calculate  $\lambda_{\max}$  and  $\lambda_{\min}$  for the detection of the primary signal. This way, the eigenvalues can be obtained by simple algebraic operations. This method reduces computational complexity since the eigenvalue decomposition processing is avoided. It is well known that the power method is an effective method to compute the maximum eigenvalue and the corresponding eigenvector (commonly referred to as maximum eigenvector) for a real-valued matrix  $B$ . It is important to note that this method is still very effective even if  $B$  is a complex valued matrix. To get a more precise result, the minimum eigenvalue  $\lambda_{\min}$  of  $R$  is computed as follows

$$\lambda_{\min} = \frac{\text{tr}(R) - \lambda_{\max}}{M - 1}$$

Where  $\text{tr}(R)$  represents the trace of  $R$ . Finally, the test statistic of the optimal detector is obtained:  $= \lambda_{\max} / \lambda_{\min}$

## 3. Computational Complexity

Here, the computational complexity of the power method and eigenvalue decomposition when computing eigenvalues is briefly investigated.  $O(n^3)$  is used to represent the order of  $n^3$  multiplications. The eigenvalue decomposition processing solves for the complete set of eigenvalues and eigenvectors of the matrix even if the problem requires only a small subset of them to be computed. For the  $n \times n$  matrix  $B$ , eigenvalue decomposition calls for  $2n^3 (t+1)$  real multiplications where  $t$  is the maximum number of iterations required to reduce a super-diagonal element as to be considered zero by the convergence criterion [14]. Thus the computational complexity of eigenvalue decomposition is  $O(n^3)$ . The idea of the power method

is only to compute the principal eigenvalues and eigenvectors. The method only consists of two main computational steps:

- Obtaining the iteration vector  $v_k$  by computing  $v_k = Bv_{k-1}$
- Computing  $v_k = v_k / \|v_k\|$  in (10)

Since  $v_k$  is an  $n \times 1$  vector, the computation of these two steps calls for  $4n^2$  and  $4n$  real multiplications, respectively. Suppose the number of iterations is  $S$ , then the total number of real multiplications is  $4S(n^2 + n)$ , that is, the computational complexity of the power method has a lower computational complexity than the eigenvalue decomposition processing when computing eigenvalues.

## 4. Threshold Definition

In the general model of the spectrum sensing algorithm, a threshold must be determined to compare with a test statistic of the sensing metric in order to sense the presence of a primary user. Consequently, to find the threshold for the statistical test, it is important to study the statistical distribution of the covariance matrix. The eigenvalue distribution of  $R$  is very complicated [15]. Moreover, there is little or no information about the signal. In fact, it is difficult to know whether the signal is present or not. This in turn makes the choice of the thresholds very difficult. Therefore, in this subsection, random matrix theory is used to approximate the distribution of this random variable and derive the decision threshold based on the pre-defined probability of false alarm, PFA. When the primary signal is absent,  $R$  turns to  $R_w$ , the covariance matrix of the noise defined as  $R_w = \frac{1}{N_s} \sum_{n=m}^{M-1+N_s} w(n)w^H(n)$ .  $R_w$  is nearly a Wishart random matrix [15].

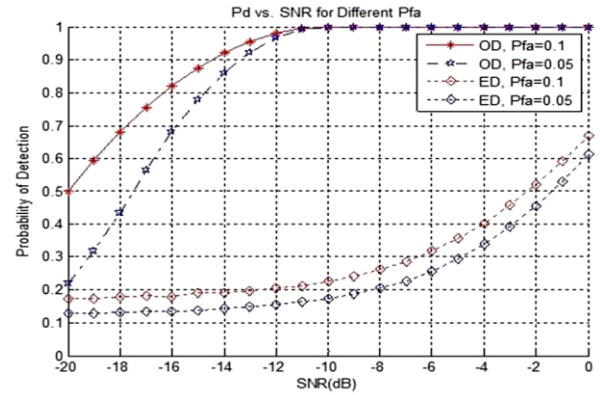


Figure - 3. Probability of Detection versus SNR for Different Probability of False Alarm

In recent years, the study of the eigenvalue distributions of a random matrix has become a very hot topic in the fields of mathematics, communication and even physics. The joint probability density function (PDF) of ordered eigenvalues of a Wishart random matrix has been known for many years [15]. However, since the expression for the PDF is very complicated, no closed form expression has been found for the marginal PDF of ordered eigenvalues. Recently though, researchers have found the distribution of the largest eigenvalue [10] and smallest eigenvalue [8] for real and complex matrices.

## 5. Simulations

In the following section, some simulation results are given using randomly generated signals to illustrate the performance of the

proposed optimal detection method. Consider a licensed frequency band in the cognitive radio network with only one active primary user. The primary signal employs Binary Phase Shift Keying (BPSK) modulation and the center frequency is 8 MHz. the sampling rate is set at 32 MHz.  $N_s$  is the number of samples and  $P$  is the temporal smoothing factor. The results are averaged over 1000 tests using Monte-Carlo realizations (for each realization, random channel, random noise and random BPSK inputs are generated) written in Matlab. The SNR of a CR is defined as the ratio of the average received signal power to the average noise power over the licensed frequency band.

$$\text{SNR} = \frac{E(\|X(n) - W(n)\|^2)}{E(\|W(n)\|^2)} \quad (9)$$

The probability of false alarm is required to be  $\text{PFA} \leq 0.1$ , then the threshold is found. For Comparison, energy detection is also simulated with noise uncertainty for the same system. The threshold for energy detection is given in [3]. At noise uncertainty, the threshold is always set based on assumed/estimated noise power. Figure 3 shows the probability of detection curves for optimal-detection and Energy Detection (ED). The results are taken for  $N_s = 100000$  and SNRs varying from  $-20\text{dB}$  to  $0\text{dB}$ . In the optimal detector, the temporal smoothing factor is 8. As shown in the figure, the proposed optimal-detection method can achieve satisfactory detection performance even in low SNR conditions. For example, the optimal-detection method can detect primary user signals with 99% probability at SNR of  $-10\text{dB}$ . However, the detection probability of ED is less than 70% percent. From the figure, we can also see that for the same SNR, the probability of detection improves as probability of false alarm.

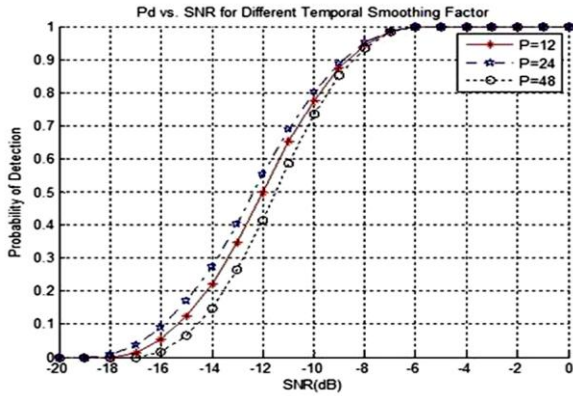


Figure - 4. Probability of Detection versus SNR for Different Temporal Smoothing

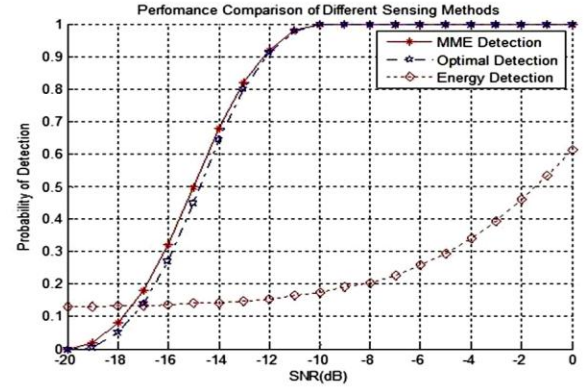


Figure - 5. Performance Comparison of Different Sensing Methods with  $\text{PFA} = 0.05$  Increases.

This reflects the trade-off between false alarm and detection probability. The probability of detection versus SNR for different temporal smoothing factors is shown in Figure 4. The results are taken for  $\text{PFA} = 0.1$ , and SNRs varying from  $-20\text{dB}$  to  $2\text{dB}$ . It is shown that the detection performance becomes better when  $P$  increases from 12 to 24. However, when  $P$  turns to 48, the performance detection declines. Therefore,  $P$  should be relatively small while using this technique for a given number of samples. Figure 5 shows the performance comparison of the optimal detection technique, the MME detection and energy detection. In MME detection, 4 receiving antennas are used for sensing in the radio environment while the optimal detector has a temporal smoothing factor of 16. For all the three methods, a probability factor of  $\text{PFA} = 0.05$  is chosen and SNR varied from  $-20\text{dB}$  to  $0\text{dB}$ . As shown on the figure, the proposed optimal detection technique performs better than the energy detection method.

Also, it can be observed that both MME detection and optimal-detection can detect the primary user signal with 100% probability when the SNR is more than  $-10\text{dB}$ . The performance of the Optimal-detection technique is very close to that of MME detection when the SNR is less than  $-10\text{dB}$ . For example, the detection probabilities of MME detection and optimal-detection are 0.820 and 0.800 at SNR of  $-13\text{dB}$  respectively. The biggest performance gap between these two methods is only 0.051 with change in SNR. In other words, the proposed optimal-detection method can achieve roughly the same performance as MME detection by using a single antenna. The main reason for this is that the processed data of these two methods have similar structures. The information about the primary user signal is perfectly contained in the data model of both methods, thus they can achieve roughly the same performance.

In summary, all the simulations show that the proposed method works well without using the information of the signal, channel and noise power. The optimal-Detection technique is always better than the energy detection method. Therefore energy detection method is not reliable since it has a low probability of detection and high probability of false alarm when there is noise uncertainty. Figure 5 shows the performance comparison of the optimal detection technique, the MME detection and energy detection. In MME detection, 4 receiving antennas are used for sensing in the radio environment while the optimal detector has a temporal smoothing factor of 16. For all the three methods, a probability factor of  $\text{PFA} =$



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## 6. Conclusions

A method based on the eigenvalues of the sample covariance matrix of the received signal has been proposed using a single antenna for cognitive radio networks. A temporal smoothing technique is utilized to form a virtual multi-antenna structure. In order to calculate the maximum and minimum eigenvalues of the covariance matrix obtained by the virtual multi-antenna structure, the proposed method uses power method. Latest random matrix theories have been used to set the decision thresholds and obtain the probability of detection in order to achieve a good detection performance. Simulations using randomly generated signals are presented in order to illustrate the performance of the Optimal-detection method. It has been shown that the performance of optimal detection is very close to that of the MME detection with multiple antennas. The method can be used for various signal detection applications without knowledge of signal, channel and noise power. Besides, the proposed optimal-detection method can reduce system overhead and avoid the eigenvalue decomposition processing by utilizing power method. The energy detector is known for its simplicity of implementing and low complexity. However, its weakest point is that it is not effective under the condition that SNR is an unknown, consequently leading to its unguaranteed accuracy. The eigenvalue-based technique on the other hand, is not as stable as the cyclo stationary technique since its threshold varies greatly as it needs to solve the problem of appropriately estimating the size of the covariance matrix. The advantage of the optimal detector, however, is that it does not

require knowledge of the primary user signal and performs better than the energy detector.

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