Review paper

A Comparative Study of New Methods with Existing Method for Solving “Assignment Problem”.

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Abstract: Assignment problem is an important problem in mathematics and example of the combinatorial optimization problem and also discuss in the real physical world. In this paper propose new methods to solve the Assignment problem and performs a comparative analysis of different methods to solve the Assignment problem. The most attractive features of these methods are that requires very simple arithmetical and logical calculations. The paper explains the Assignment problem with an example using new methods and computes by existing one method. Also, we compare the optimal solutions among these new methods and existing method. The new methods are the systematic procedure, easy to apply for solving assignment problem.

Keywords: Assignment problem, Hungarian Assignment method, new methods, cost, matrix, optimization.

1. Introduction

Let there is only operator and machine in the printing press. How would you employ the worker? Your immediate answer will be, the available operator will operate the machine. Again suppose there are two machines in the press and two operators are engaged at different rates to operate them. Which operator should operate which machine for maximizing profit? Similarly, if there are 'n' machines available and 'n' persons are engaged at different rates to operate them. Which operator should be assigned to which machine to ensure maximum efficiency? While answering the above questions we have to think about the interest of the press, so we have to find such an assignment by which the press gets maximum profit on minimum investment. Such problems are known as “Assignment problems".

2. Formulation of the assignment problem

Let there are 'n' jobs and 'n' persons are available with different skills . If the cost of doing j^{th} work by i^{th} person is C_{ij} . Then **the cost matrix** is given the table -1 below.

Table – 1

Jobs / Persons	1	2	3	--------j	--------n
1	C_{11}	C_{12}	C_{13}	--------C_{1j}	--------C_{1n}
2	C_{21}	C_{22}	C_{23}	--------C_{2j}	--------C_{2n}
3	C_{31}	C_{32}	C_{33}	--------C_{3j}	--------C_{3n}
⋮ i	⋮ C_{i1}	⋮ C_{i2}	⋮ C_{i3}	⋮ --------C_{ij}	⋮ --------C_{in}
⋮ n	⋮ C_{n1}	⋮ C_{n2}	⋮ C_{n3}	⋮ --------C_{nj}	⋮ --------C_{nn}

Presently the issue is which work is to be allotted to whom with the goal that the cost of finish of work will be mi. mathematically, we can express the problem as follows

To minimize

$z(\cos t) = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$; $where\ i = 1,2,---n, j = 1,2,---n \rightarrow (1)$

When

$x_{ij} = \begin{cases} 1;\ if\ i^{th}\ person\ is\ assigned\ to\ j^{th}\ work \\ 0;\ if\ i^{th}\ \text{person is not assigned to j}^{\text{th}}\ \text{work with the restrictions} \end{cases}$

$\sum_{i=1}^{n} x_{ij} = 1, j = 1,2,---n, i.e., i^{th}$ person will do only one work

$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2,---n, i.e.\ j^{th}$ work will be done onlyby one person

3. Solution of the Assignment Problem

So far, in the literature it is available that an assignment problem can be solved by the following four methods.

- Enumeration method
- Simplex method
- Transportation method
- Hungarian method

Here, we discuss Hungarian method and new methods

4. Assignment Algorithm for Hungarian Method

Phase - 1: Row and column reductions

Step – 1: Subtract the minimum value of each row from the entries of that row.
Step – 2: Subtract the minimum value of each column from the entries of the column.

Phase - 2: Optimization of the problem

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Step – 1: Draw minimum number of lines to cover the all zeros of the matrix.

Procedure:

(a) Row Scanning:

- Starting from the first row, ask the following question. Is there exactly one "zero" in that row? If Yes, mark a square around that zero entry and draw a vertical line passing through that zero ; otherwise skip that row .
- After scanning the last row, check whether all the zeros are covered with lines. If yes, go to

Step – 2: otherwise, do column scanning.

(b) Column Scanning

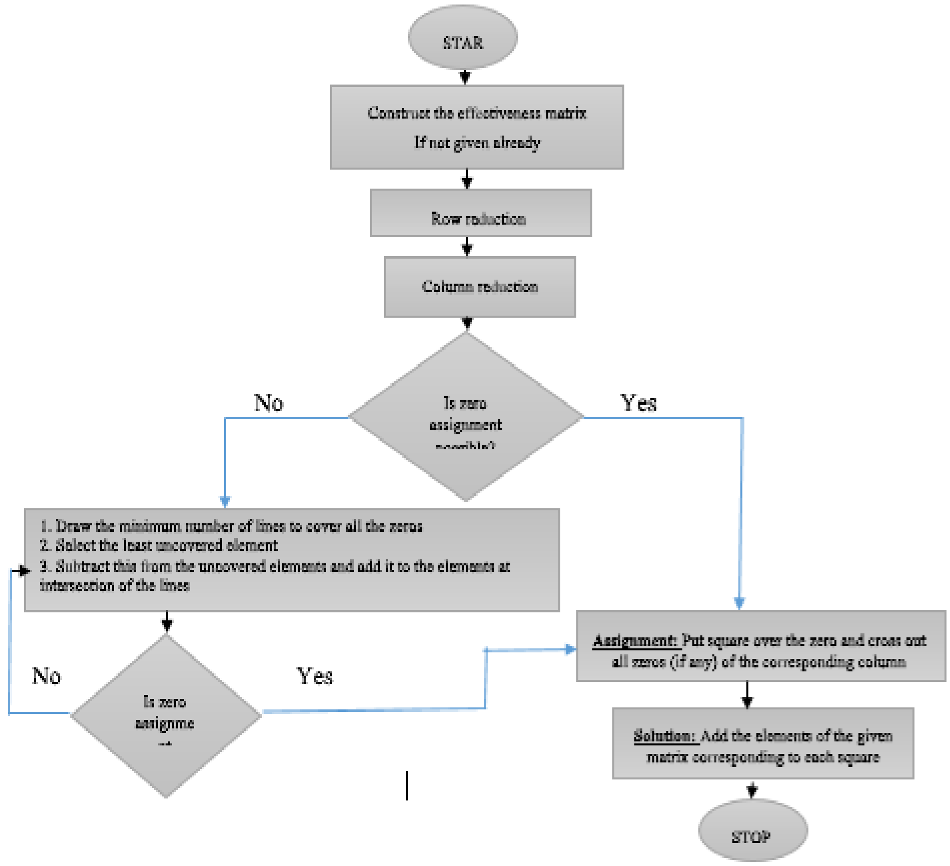
- Start from the first column, ask the following question. Is there exactly one zero in that column? If Yes mark a square around that zero entry and draw a horizontal line passing through that zero ; otherwise skip that column .
- After scanning the last column, check whether all the zeroes are covered with lines.

Step - 2: Check whether the number of squares marked is equal the number of rows of the matrix. If yes, go to step - 5; otherwise go to next step.
Step - 3: Identify the minimum value of the undeleted cell values.

- Add the minimum undeleted cell value at the intersection points of the present matrix.
- Subtract the minimum undeleted cell value from all the undeleted cell values.
- All other entries remain same.

Step - 4: Go to step -1
Step - 5: Treat the solution as marked by the squares as the optimal solution

4.1: Flow chart for Hungarian method



Flow chart -1:

5. New Methods for Solving Assignment Problem

The new alternate methods of assignment problem discussed here gives optimal solution directly within few steps. It is very easy to calculate and understand. The alternate methods developed by[14]and [9] us in this investigation seems to be easiest as compare to available methods of assignment problem. Here we explain algorithm for alternate method of solving assignment problem for minimization. Now we consider the assignment the matrix when C_{ij} is the cost of assigning i^{it} job to jth machine?

Table- 2:

	1	2	3	- - - - - n
1	C_{11}	C_{12}	C_{13}	- - - - - C_{1n}
2	C_{21}	C_{22}	C_{23}	- - - - - C_{2n}
3	C_{31}	C_{32}	C_{33}	- - - - - C_{3n}
⋮	⋮	⋮	⋮	⋮
n	C_{n1}	C_{n2}	C_{n3}	- - - - C_{nn}

5.1. Assignment Algorithm for New Method-I

Step – 1: Subtract the minimum value of each row from the entries of that row

Step – 2: Now add 1 to all elements and get at least one ones in each row. Then make assignment in terms of ones. If there are some rows and columns without assignment, then we cannot get optimum solution. Then we go to the next step.

Step – 3: Draw the minimum number of lines to cover all the 'ones', of the matrix.

Procedure

5.1.1 Row scanning

Starting from the first row, ask the following question. Is the exactly only single "one "in that row? If yes, mark a square around that "one "entry and draw a vertical line passing through that "one"; otherwise skip that row.

After scanning the last row, check whether all 'ones' are covered with lines. If yes, then we get optimal solution; Otherwise do column scanning.

5.1.2. Column Scanning

Starting from the first column, ask the following question. Is there exactly single "one" in that column? If Yes , mark a square around that " one " entry and draw a horizontal line passing through that " one "; otherwise skip the column.

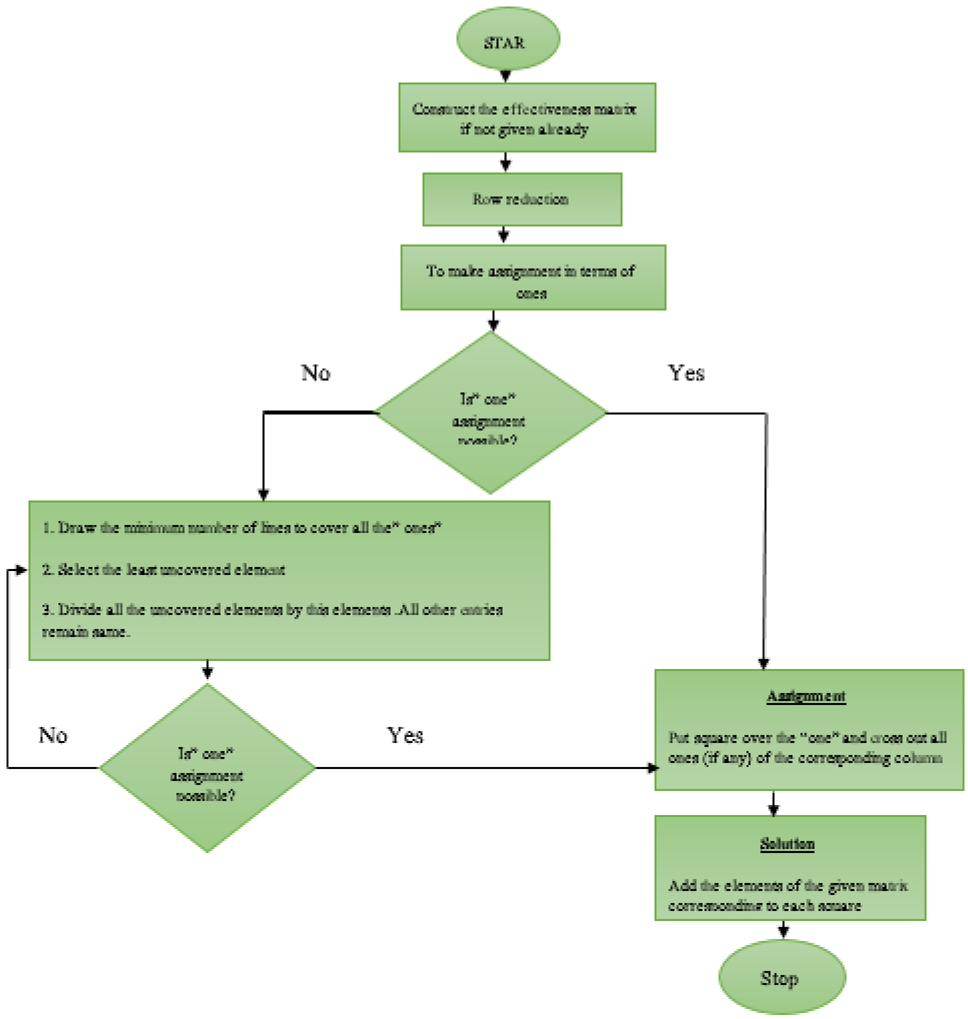
- After scanning the last column , check whether all the ' ones ' are covered with lines.

Step – 4: Check whether number of squares marked is equal to number of rows of the matrix .Then we get optimal solution; Otherwise go to next step.

Step – 5: Identify the minimum value of the undeleted cell values. Divide all undeleted cell values by this minimum value. All other entries remain same. Then we get some new 'ones' in row and column . Again make assignment in terms of ones.

Step – 6: If we cannot get the optimal assignment in each row and column . Then repeat steps – (3) and (4) successively till an optimum solution is obtained.

5.1.3. Flow chart for Assignment problem (Proposed method-I)



5.2. Assignment Algorithm for New Method-I

Let A, B, C … Z denote resources and I, II, III, IV… denote the activities. Now we discussed various steps for solving assignment problem which are following.

Step – 1 : Construct the data matrix of the assignment problem. Think about row as a laborer (asset) and section as a vocation (action).

Step – 2 **:** Write two sections, where segment 1 speaks to asset and segment 2 speaks to a movement. Under segment 1, compose the asset, say, A, B, C … Z. Next discover least unit cost for each line, whichever least esteem is accessible in the regarding segment, select it and compose it in term of exercises under segment 2.Continue this procedure for all the Z pushes and write in term of I, II …

Step – 3: Let for every asset; if there is exceptional movement at that point appointed that action for the relating asset, consequently we accomplished our ideal arrangement.For **example**, let we have 5 resources A, B, C, D, E and 5 activities I, II, III, IV, V. This is shown in Table 3

Table – 3:

Resource(Employs)	Activity(Jobs)
A	V
B	III
C	I
D	IV
E	II

If there is no unique activity for corresponding resources then the assignment can be made using following given steps:

Step – 4: Look at which of any one asset has exceptional movement and after that allot that action for the relating asset. Next erase that line and its comparing segment for which asset has just been appointed.

Step – 5: Again locate the base unit cost for the rest of the lines. Check on the off chance that it fulfill stage 4 at that point perform it. Something else, check which columns have just a single same movement. Next discover distinction amongst least and next least unit cost for every one of those lines which have same movement. Erase those lines and comparing sections for which those assets have been relegated.

Remark 1 However in the event that there is tie in distinction for two and in excess of two movement at that point additionally take the contrast amongst least and beside next least unit cost. Next check which action has most extreme distinction, relegate that movement.

Step – 6: Repeat steps 4 to 5 till all jobs are assigned uniquely to the corresponding activity.

Step – 7: Once all the jobs are assigned then calculate the total cost by using the expression,

Total cost = $\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

6. Numerical Comparison of Existing Method with Proposed New Methods

In this paper, we illustrate the numerical example to solve the assignment problem using New Alternate Methods and Hungarian method.

6.1. Solve the following assignment problem using New Method-I

Consider the following Assignment problem .The department has five employees with five jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix.

Table – 4:

Employees	Jobs				
	I	II	III	IV	V
A	10	6	4	8	3
B	2	11	7	7	6
C	5	10	11	4	8
D	6	5	3	2	5
E	11	7	10	11	7

How does jobs be allocated, one per employee, so as to minimize the total man-hours?

Solution

Step – 1: Subtract the minimum value of each row from the entries of that row. Then the reduced matrix as follows

Table – 5:

Employees	Jobs				
	I	II	III	IV	V
A	7	3	1	5	0
B	0	9	5	5	4
C	1	6	7	0	4
D	4	3	1	0	3
E	4	0	3	4	0

Step – 2: Now add 1 to all elements

Table – 6:

Employees	Jobs				
	I	II	III	IV	V
A	8	4	2	6	1
B	1	10	6	6	5
C	2	7	8	1	5
D	5	4	2	1	4
E	5	1	4	5	1

Step – 3: Now make initial assignment

Table – 7:

Employees	Jobs				
	I	II	III	IV	V
A	8	4	2	6	1
B	1	10	6	6	5
C	2	7	8	1	5
D	5	4	2	1	4
E	5	1	4	5	4

Here the number of squares≠Number of rows of the matrix (i.e., 4≠5)

Step - 4: Identify the minimum value of the undeleted cell values. Divide all undeleted cell values by this minimum value. All other entries remain same. Then make assignment again.

Table – 8:

Employees	Jobs				
	I	II	III	IV	V
A	8	4	1	6	1
B	1	10	3	6	5
C	2	7	4	1	5
D	5	4	1	1	4
E	5	1	2	5	1

Here the number of squares=Number of rows of the matrix=5.So we get optimal solution.

The solution is $(A, V), (B, I), (C, IV), (D, III), (E, II)$.

Therefore total cost=Add the elements of the given matrix corresponding to each square. $= 3 + 2 + 4 + 3 + 7 = 19$

6.2. Solve the following assignment problem using proposed Method-II

Consider the following Assignment problem. The department has five employees with five jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix.

Table – 9:

Employees	Jobs				
	I	II	III	IV	V
A	10	6	4	8	3
B	2	11	7	7	6
C	5	10	11	4	8
D	6	5	3	2	5
E	11	7	10	11	7

How the jobs should be allocated, one per employee, so as to minimize the total man-hours.

Solution

Solution Consider above Table-1, Select row A, where the minimum value is 3 representing job V. Similarly, the minimum value for row-B to row-E are 2, 4, 2, and 7 representing jobs I, IV, IV and II, V respectively. This is shown in Table 1

Table – 9.1:

Resource(Employee)	Activity(Jobs)
A	V
B	I
C	IV
D	IV
E	II,V

Table - 9.1a:

Employees	Jobs				
	I	II	III	IV	V
A	10	6	4	8	3
B	2	11	7	7	6
C	5	10	11	4	8
D	6	5	3	2	5
E	11	7	10	11	7

Here activity I unique as it doesn't occur again and hence assigned resource B to activity I and is shown in table 1.1a. .Next delete Row B and Column I. Again select minimum cost value for the remaining resources, A, C, D and E. which is shown below Table 1.2

Table - 9.2

Resource(Employee)	Activity(Jobs)
A	V
C	IV
D	IV
E	II,V

Table - 9.2a

Employees	Jobs			
	II	III	IV	V
A	6	4	8	3
C	10	11	4	8
D	5	3	2	5
E	7	10	11	7

Since resource A has single activity V. Next we see that resource E has also activity V and hence we take the minimum unit cost difference for resource A and E. Here minimum cost difference for resource A is 1(4-3) while minimum cost difference for resource E is 0(7-7). Since 1 is the maximum difference which represents resource A and hence assign resource A to activity V and is shown in Table 9.2a. Further delete row A and Column. Again select minimum unit cost for the remaining resources, C, D and E, Which is shown in Table 9.3.

Table - 9.3

Resource(Employee)	Activity(Jobs)
C	IV
D	IV
E	II,V

Table - 9.3a

Employees	Jobs		
	II	III	IV
C	10	11	4
D	5	3	2
E	7	10	11

Since resource C and D have single activity IV then we take the minimum cost difference for resources C and D. Here minimum cost difference for resource C is 6(10-4), minimum cost difference for resource D is 1(3-2). Since 6 is a maximum difference which represents resource C and hence assign resource C to activity IV and is shown in Table 1.3a. Further delete row C and Column IV. Again select minimum cost value for the remaining resources, D and E. which is shown in Table 1.4

Table -9.4:

Resource(Employee)	Activity(Jobs)
C	IV
D	IV
E	II,V

Table – 9.4a:

Employees	Jobs	
	II	III
D	5	3
E	7	10

Since resource D has single activity III and E has single activity II then assign resource D to activity III and assign resource E to activity II and is shown in Table 1.4a .Finally, different employees have assigned jobs uniquely, which is shown in Table 1.5.

Table – 9.5:

Resource (Employees)	Activity(Jobs)	Time(hours)
A	V	3
B	I	2
C	IV	4
D	III	3
E	II	7
	Total	19

6.3. Solve the following assignment problem using Hungarian Method

Consider the following Assignment problem .The department has five employees with five jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix.

Table – 9.6:

Employees	Jobs				
	I	**II**	**III**	**IV**	**V**
A	10	6	4	8	3
B	2	11	7	7	6
C	5	10	11	4	8
D	6	5	3	2	5
E	11	7	10	11	7

How should the jobs be allocated, one per employee, so as to minimize the total man-hours?

Step – 1: Subtract the minimum value of each row from the entries of that row. Then the reduced matrix as follows

Table – 9.7:

Employees	Jobs				
	I	**II**	**III**	**IV**	**V**
A	7	3	1	5	0
B	0	9	5	5	4
C	1	6	7	0	4
D	4	3	1	0	3
E	4	0	3	4	0

Step – 2: Subtract the minimum value of each column from the entries of that column. Then the reduced matrix as follows

Table – 9.8:

Employees	Jobs				
	I	II	III	IV	V
A	7	3	0	5	0
B	0	9	4	5	4
C	1	6	6	0	4
D	4	3	0	0	3
E	4	0	2	4	0

Step (3): Now make initial assignment

Table – 9.9:

Employees	Jobs				
	I	II	III	IV	V
A	7	3	0	5	0
B	0	9	4	5	4
C	1	6	6	0	4
D	4	3	0	0	3
E	4	0	2	4	0

Here the number of squares=Number of rows of the matrix=5.So we get optimal solution. The solution is $(A,V),(B,I),(C,IV),(D,III),(E,II)$.

Therefore total cost=Add the elements of the given matrix corresponding to each square. $= 3 + 2 + 4 + 3 + 7 = 19$

Here new methods are introduced for solving Assignment problem. Also an example using the New Methods and Hungarian method is examined and the optimal solutions are compared among three methods and the optimal solutions by three methods are same

7. Comparison of optimal values of three methods

Table – 10:

Example	HA-method	New method-I	New method-II	Optimum
01	19	19	19	19

8. Conclusion

In this paper, we presented (reviewed) new methods for solving Assignment problem. Initially we explained the new methods and showed the efficiency of it by numerical example . And we get the optimal solution which is same as the optimal solution of HA-method therefore this paper introduces a different approach which is easy to solve Assignment problem.

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